MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017

TMT 1171 – MATHEMATICAL TECHNIQUES I

(All sections / Groups)

7 OCTOBER 2016
9.00 A.M. – 11.00 A.M.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **5 pages** only including the cover page with **5 Questions** and an **Appendix**.

2. There are **2 Sections**:
   Section A (3 Questions): Attempt **ALL THREE (3)** questions.
   Section B (2 Questions): Attempt **ONE (1)** question **ONLY**.
   All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write your answers in the Answer Booklet provided, and **start each question on a new page**.

4. Show all steps.

5. Formulas are provided and attached in Appendix.

SECTION A

Answer ALL THREE (3) Questions.

Question 1

a. Find the angle between the vectors $u = i - 2j + 2k$ and $v = -3i + 2j + k$. **[2 marks]**

b. Consider the points $P = (-1, 2, 1)$, $Q = (0, -3, 2)$, and $R = (1, 1, -4)$.

i. Find the vectors $u = \vec{PQ}$ and $v = \vec{PR}$. **[3 marks]**

ii. Compute the cross product of $\vec{PQ}$ and $\vec{PR}$. **[2 marks]**

c. Let K be the line passing through $A(5,0,-2)$ that is parallel to the line joining $B(2,3,-1)$ and $C(-1,0,1)$. Determine the parametric equation of the line K. **[3 marks]**

Question 2

a. Solve the following linear system by using Cramer's Rule.

$$z + x = 8$$
$$y + x + 2z = 17$$
$$x + 2y + z = 16$$

[4 marks]

b. The following linear system

$$2x + 6y = 1$$
$$x - 3y = 3$$

can be written in the form of $AX = B$, i.e.

$$\begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

i. Write down the characteristics equation of A. **[2 marks]**
ii. From (i), find the eigenvalues. **[2 marks]**
iii. Then, find the corresponding eigenvectors. **[2 marks]**

Continued.....

Question 3

a. Given the complex numbers $z = 1 - i$ and $w = 2\sqrt{3} - 2i$.

i. Write z and w in polar form. **[5 marks]**

ii. Using the result of in (i), find the product of zw in polar form. **[2 marks]**

b. Let z_1 and z_2 be two complex numbers given as $z_1 = 2 + i$ and $z_2 = 3 - 2i$. Solve the followings:

i. $z_1\overline{z_1}$ **[1.5 marks]**

ii. $z_1 z_2$ **[1.5 marks]**

SECTION B

Answer ONE (1) Question ONLY.

Question 4

a. i. Given $f(x) = \begin{cases} 6x - 1, & x < 2 \\ 4x + 3, & x \geq 2 \end{cases}$

Compute $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$. Hence, determine whether $\lim_{x \to 2} f(x)$ exists by applying the suitable limit theorem(s) or rule(s). **[3 marks]**

ii. By applying the appropriate technique(s) or rule(s) for limit, compute $\lim_{x \to 3} x^2 - 2x + 10$.

[2 marks]

b. By using the appropriate technique(s), find

i. the second derivative (y'') of $y = -x^{-2} + 5x - e^x$. **[2 marks]**

ii. $\frac{dy}{dx}$ for $xy + \ln x = 0$. **[3 marks]**

Continued.....

Question 5

a. Apply the appropriate technique(s) or rule(s) for limit, compute

i. $\lim_{x \to -1} 2x^3 + 5x$ **[1.5 marks]**

ii. $\lim_{x \to 0} \frac{e^x}{2^x}$ **[1.5 marks]**

iii. $\lim_{x \to 3} (x+3)(x^2 - 3x)$ **[2 marks]**

b. i. Use the *Fundamental Theorem of Calculus* to compute $\int_{-1}^{0} (2x^3 - 2x + 3)dx$. **[2 marks]**

ii. Use *integration by substitution* to compute $\int 3x^2 \sqrt{1+x^3}\, dx$. **[3 marks]**

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APPENDIX

1. Length of a vector v : $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$
2. $\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos\theta$
3. Dot product: $\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$
4. Parametric equation: $x = x_0 + at$; $y = y_0 + bt$; $z = z_0 + ct$
5. Polar form of complex number ($z = x+yi$): $z = r(\cos\theta + i\sin\theta)$, where $r = \sqrt{x^2 + y^2}$
6. Derivative Formulas

$\frac{d}{dx} e^x = e^x$
$\frac{d}{dx} \ln x = \frac{1}{x}$
Power Rule: $\frac{d}{dx} x^n = nx^{n-1}$ if $f(x) = x^n$ with $n \in R$
Product Rule: $\frac{d}{dx}(f \cdot g) = f\frac{dg}{dx} + g\frac{df}{dx}$

7. Integration Formulas

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, for $n \neq -1$, n rational
Integration by Substitution: If $u = g(x)$ and $du = g'(x)dx$, then $\int f(u)du = \int f(g(x))\frac{d}{dx} g(x)dx$

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